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## Magnetization of layered Heisenberg ferrimagnets

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**Abstract.** The method of the double-time–temperature Green function is used to study theoretically the magnetic properties of layered Heisenberg ferrimagnets. The sublattice magnetization is calculated for different interlayer coupling strengths in the whole range of temperatures. According to the extent to which the interlayer coupling suppresses the two-dimensional spin fluctuations, we divide the low-temperature regime into two parts and give the asymptotic expressions for sublattice magnetization in two low-temperature regimes and in the vicinity of the Curie temperature, respectively. We also discuss the dependence of the Curie temperature on the interlayer coupling strength.

### 1. Introduction

Layered magnetic systems have been a subject of growing interest in recent years. The discovery of copper oxide high- $T_c$  superconductors with quasi-two-dimensional (quasi-2D) magnetic properties in their parent materials greatly stimulates further studies in this field. Theoretically, many investigations on layered magnetic systems have been focused on the layered ferromagnets and antiferromagnets. It is well established that a pure 2D Heisenberg system does not achieve long-range ordering (LRO) at a finite temperature [1], whereas a quasi-2D system can achieve LRO at a non-zero temperature with the interlayer coupling suppressing 2D spin fluctuations [2–16]. At low temperatures, when the interlayer coupling is weak, the properties of a system undergo dimensional crossover with an increase in temperature from zero. For example, in layered ferromagnets the magnetization changes from a  $T^{3/2}$  to a  $T \ln T$  behaviour [10, 11], and in layered antiferromagnets the sublattice magnetization changes from a  $T^2$  to a  $T \ln T$  behaviour [3, 12–14].

Practical ferrimagnets generally possess rather complicated lattice structures and hence are difficult to handle. In order to retain the basic feature of ferrimagnets, theoretically a two-sublattice model is often used [17–22]. For homogeneous ferrimagnets, Nakamura and Bloch [17] investigated the temperature dependence of magnon frequencies using the Holstein–Primakoff transformation. Lin and Zhang [19] investigated the spin-wave spectrum and sublattice magnetization of Heisenberg ferrimagnets at  $T = 0$  K using the method of the retarded Green function equation of motion. Xue *et al* [21] investigated the spin-wave spectrum and sublattice magnetization of anisotropic ferrimagnets using the same method as in [19].

In this paper, we shall study the magnetization behaviour of layered Heisenberg ferrimagnets in the whole range of temperatures; it is a natural expansion of the previous

work on its low-temperature behaviour [22]. The paper is arranged as follows: in section 2, we start with a layered ferrimagnet Heisenberg model and then give the fundamental equations. In section 3, we give the numerical results of sublattice magnetization in the whole range of temperatures, the asymptotic expressions of the sublattice magnetization in different temperature regimes, the calculation formula of the Curie temperature, and so on. The last section 4 is devoted to the summary.

## 2. Fundamental equations

We consider a layered Heisenberg ferrimagnetic model on a simple-cubic lattice with intralayer lattice parameter  $a$  and interlayer lattice parameters  $a$  and  $c$ . Suppose that the lattice is divided into two sublattices A and B,  $i \in A$  and  $j \in B$ , and the spin values are  $S_a$  in sublattice A and  $S_b$  in B; there are  $N$  sites on each sublattice [22]. The Hamiltonian is

$$H = \sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \quad (1)$$

where the summation is taken over all nearest-neighbour sites  $\langle i, j \rangle$ . For convenience, we define  $J_{ij}$  as

$$J_{ij} = \begin{cases} J & \text{if sites } i \text{ and } j \text{ are in the same layer} \\ J_{\perp} & \text{if sites } i \text{ and } j \text{ are in two nearest-neighbour layers.} \end{cases} \quad (2)$$

To analyse the layered Heisenberg ferrimagnets, we introduce the following Green functions according to Callen [23]:

$$G_A(E, i_1, i_2) = \langle\langle S_{i_1}^+ | \exp(pS_{i_2}^z) S_{i_2}^- \rangle\rangle_E \quad (3a)$$

$$F_A(E, j_2, i_2) = \langle\langle S_{j_2}^- | \exp(pS_{i_2}^z) S_{i_2}^- \rangle\rangle_E \quad (3b)$$

$$G_B(E, j_1, j_2) = \langle\langle S_{j_1}^+ | \exp(qS_{j_2}^z) S_{j_2}^- \rangle\rangle_E \quad (4a)$$

$$F_B(E, i_2, j_2) = \langle\langle S_{i_2}^- | \exp(qS_{j_2}^z) S_{j_2}^- \rangle\rangle_E \quad (4b)$$

where  $p$  and  $q$  are parameters. For convenience, we introduce the transformation  $S_j^z \rightarrow -S_j^z$ ,  $S_j^+ \rightarrow S_j^-$  and  $S_j^- \rightarrow S_j^+$ , as did Cheng and Pu [24]. Using the technique of the equation of motion for the Green functions, within the Tyablikov decoupling approximation, we obtain the Fourier components of the Green functions:

$$G_A(E, k) = \frac{F_A(p)}{E_A^+ - E_A^-} \left( \frac{E_A^+ + 4J(2 + \delta)\langle S^z \rangle_A}{E - E_A^+} - \frac{E_A^- + 4J(2 + \delta)\langle S^z \rangle_A}{E - E_A^-} \right) \quad (5)$$

$$G_B(E, k) = \frac{F_B(q)}{E_B^+ - E_B^-} \left( \frac{E_B^+ + 4J(2 + \delta)\langle S^z \rangle_B}{E - E_B^+} - \frac{E_B^- + 4J(2 + \delta)\langle S^z \rangle_B}{E - E_B^-} \right) \quad (6)$$

where

$$F_A(p) = \langle [S_i^+, \exp(pS_i^z) S_i^-] \rangle \quad (7a)$$

$$F_B(q) = \langle [S_j^+, \exp(qS_j^z) S_j^-] \rangle \quad (7b)$$

$$E_A^{\pm} = 2J(2 + \delta)\langle S^z \rangle_A \{ (\alpha - 1) \pm [(1 - \alpha)^2 + 4\alpha(1 - \eta_k^2)]^{1/2} \} \quad (8a)$$

$$E_B^{\pm} = 2J(2 + \delta)\langle S^z \rangle_B \{ (1 - \alpha) \pm [(1 - \alpha)^2 + 4\alpha(1 - \eta_k^2)]^{1/2} \} \quad (8b)$$

$$\eta_k = \frac{\cos(k_x a) + \cos(k_y a) + \delta \cos(k_z c)}{2 + \delta} \quad (9)$$

$$\delta = J_{\perp}/J \quad (10)$$

$$\alpha = \langle S^z \rangle_B / \langle S^z \rangle_A. \quad (11)$$

Here  $\langle S^z \rangle_A$  and  $\langle S^z \rangle_B$  are sublattice magnetizations per site (the unit is taken to be  $g\mu_B$ ) in sublattices A and B, respectively. Because  $S_a \neq S_b$  in ferrimagnets, let  $S_a > S_b$  without loss generality. From equation (8) we can see that  $E_A^+ = -E_B^-$  and  $E_B^+ = -E_A^-$ , the spin-wave spectrum has two branches. When  $k \rightarrow 0$ ,  $E_A^+ = -E_B^- \rightarrow 0$ , which represents the acoustic branch, while  $E_B^+ = -E_A^-$  remains finite and represents the optical branch [17]. When  $S_a = S_b$ , the system is an antiferromagnet, and the spin-wave spectrum takes a very simple form which is identical with that of [12].

After using the spectral theorem and Callen's [23] technique, we finally obtain the magnetization of each sublattice as

$$\langle S^z \rangle_A = \frac{(S_a - n_A)(1 + n_A)^{2S_a+1} + (S_a + 1 + n_A)n_A^{2S_a+1}}{(1 + n_A)^{2S_a+1} - n_A^{2S_a+1}} \quad (12a)$$

$$\langle S^z \rangle_B = \frac{(S_b - n_B)(1 + n_B)^{2S_b+1} + (S_b + 1 + n_B)n_B^{2S_b+1}}{(1 + n_B)^{2S_b+1} - n_B^{2S_b+1}} \quad (12b)$$

where  $n_A$  and  $n_B$  are the auxiliary functions:

$$n_A = \frac{1}{N} \sum_k \frac{1}{E_A^+ - E_A^-} \left( \frac{E_A^+ + 4J(2 + \delta)\langle S^z \rangle_A}{\exp(\beta E_A^+) - 1} - \frac{E_A^- + 4J(2 + \delta)\langle S^z \rangle_A}{\exp(\beta E_A^-) - 1} \right) \quad (13a)$$

$$n_B = \frac{1}{N} \sum_k \frac{1}{E_B^+ - E_B^-} \left( \frac{E_B^+ + 4J(2 + \delta)\langle S^z \rangle_B}{\exp(\beta E_B^+) - 1} - \frac{E_B^- + 4J(2 + \delta)\langle S^z \rangle_B}{\exp(\beta E_B^-) - 1} \right). \quad (13b)$$

Equations (8)–(13) are the fundamental equations of the sublattice magnetizations; they are very complicated and must be solved self-consistently.

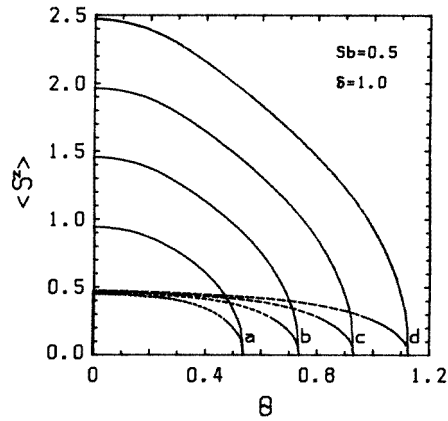
### 3. Results and discussion

First of all, we carry out numerical calculation to solve self-consistently the fundamental equations (8)–(13) about  $\langle S^z \rangle_A$  and  $\langle S^z \rangle_B$  for different sets of parameters  $S_a$ ,  $S_b$  and  $\delta$  and hence obtain the temperature dependence of the spin-wave spectra. The results of sublattice magnetizations versus reduced temperature  $\Theta = k_B T/6J$  are plotted in figures 1–4, which correspond to  $\delta = 1, 0.1, 0.001$  and  $0.00001$ , respectively. In these figures, the solid curves represent the magnetizations  $\langle S^z \rangle_A$  of sublattice A, and the broken curves represent  $\langle S^z \rangle_B$ , the minus values of the magnetizations of sublattice B. Four groups of curves a, b, c and d correspond to  $S_a = 1, \frac{3}{2}, 2$  and  $\frac{5}{2}$ , respectively. From these figures we see that, for fixed  $\delta$  and  $\Theta$ , both  $\langle S^z \rangle_A$  and  $\langle S^z \rangle_B$  increase when  $S_a$  increases; for fixed  $S_a$  and  $\delta$ , both  $\langle S^z \rangle_A$  and  $\langle S^z \rangle_B$  decrease when  $\Theta$  increases, but  $\langle S^z \rangle_A$  drops more rapidly than does  $\langle S^z \rangle_B$ . We can see also that the transition temperature, the Curie temperature, increases with increasing  $\delta$  for fixed  $S_a$  and  $S_b$ .

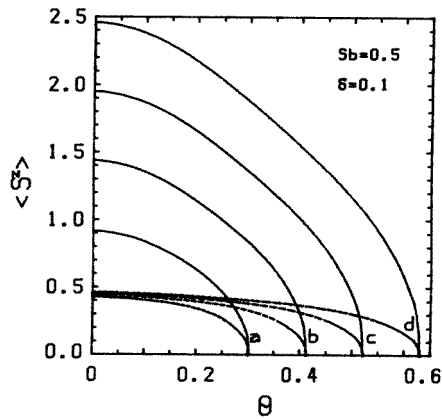
We now discuss the behaviour of the sublattice magnetizations in different temperature regimes.

At  $T = 0$  K, the factor  $[\exp(\beta E) - 1]^{-1}$  is zero for  $E > 0$  and is  $-1$  for  $E < 0$ . The auxiliary functions are then from equation (13)

$$n_{A0} = \frac{1}{N} \sum_k \frac{E_A^- + 4J(2 + \delta)\langle S^z \rangle_A}{E_A^+ - E_A^-} \quad (14a)$$



**Figure 1.** The  $\langle S^z \rangle_A$  (—), the magnetizations of sublattice A and the  $\langle S^z \rangle_B$  (---), the minus magnetizations of sublattice B, as functions of reduced temperature  $\Theta$  with  $S_b = \frac{1}{2}$  and  $\delta = 1.0$ . The values of  $S_a$  corresponding to four groups of curves a, b, c and d are 1,  $\frac{3}{2}$ , 2 and  $\frac{5}{2}$ , respectively.



**Figure 2.** The sublattice magnetizations  $\langle S^z \rangle_A$  and  $\langle S^z \rangle_B$  versus  $\Theta$ ; the parameters are the same as in figure 1, but  $\delta = 0.1$ .

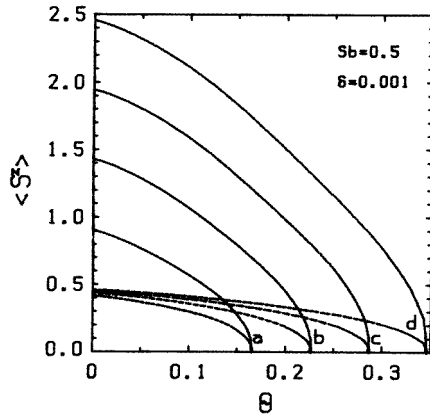
$$n_{B0} = \frac{1}{N} \sum_k \frac{E_B^- + 4J(2 + \delta)\langle S^z \rangle_B}{E_B^+ - E_B^-} \quad (14b)$$

noting that  $[E_A^- + 4J(2 + \delta)\langle S^z \rangle_A] = [E_B^- + 4J(2 + \delta)\langle S^z \rangle_B]$  and  $E_A^+ - E_A^- = E_B^+ - E_B^-$ ; thus  $n_{A0} = n_{B0}$ , which is different from  $n_{A0} + n_{B0} = -1$  in [19]; this results from the transformation of spin operators in sublattice B.

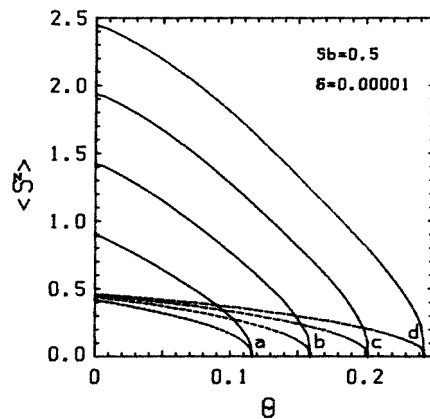
Substituting (14) into (12), we obtain the sublattice magnetizations at  $T = 0$  K as follows:

$$\langle S^z \rangle_{A0} = \frac{(S_a - n_{A0})(1 + n_{A0})^{2S_a+1} + (S_a + 1 + n_{A0})n_{A0}^{2S_a+1}}{(1 + n_{A0})^{2S_a+1} - n_{A0}^{2S_a+1}} \quad (15a)$$

$$\langle S^z \rangle_{B0} = \frac{(S_b - n_{B0})(1 + n_{B0})^{2S_b+1} + (S_b + 1 + n_{B0})n_{B0}^{2S_b+1}}{(1 + n_{B0})^{2S_b+1} - n_{B0}^{2S_b+1}}. \quad (15b)$$



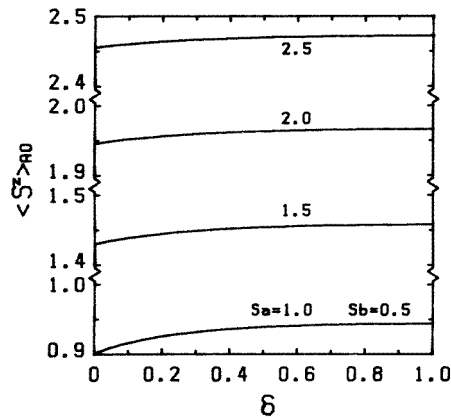
**Figure 3.** The sublattice magnetizations  $\langle S^z \rangle_A$  and  $\langle S^z \rangle_B$  versus  $\Theta$ ; the parameters are the same as in figure 1, but  $\delta = 0.001$ .



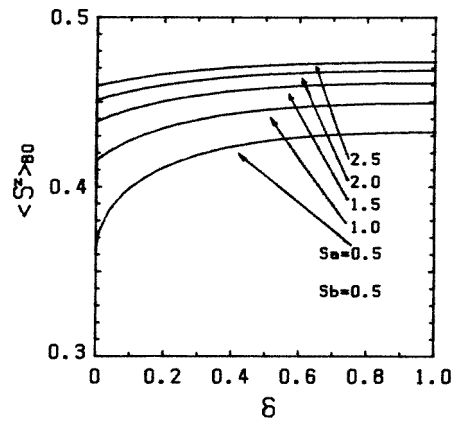
**Figure 4.** The sublattice magnetizations  $\langle S^z \rangle_A$  and  $\langle S^z \rangle_B$  versus  $\Theta$ ; the parameters are the same as in figure 1, but  $\delta = 0.00001$ .

Using equations (8)–(11), we can also solve the coupled equations (14) and (15) for different sets of parameters  $S_a$  and  $S_b$  to get the relation between sublattice magnetizations and the interlayer coupling strength at zero temperature; the results are shown in figure 5 and figure 6. We can see that the sublattice magnetizations at zero temperature are smaller than their classical values owing to the zero-point quantum fluctuations of spin. The sublattice magnetizations increase with increasing  $\delta$  for fixed  $S_a$ , and with increasing  $S_a$  for fixed  $\delta$ . At  $T = 0$  K, the magnetic property of ferrimagnets is similar to that of antiferromagnets.

In the low-temperature regime, when investigating the asymptotic forms of the sublattice magnetizations, we can neglect the contribution from the optical branch of the spectra. As in [3, 10, 12, 22], introduce  $\Theta_1 = (2 + \delta)/3$  to define the low-temperature regime  $\Theta_1 \gg \Theta$ ; thus, when the interlayer coupling is weak ( $\delta \ll 1$ ), the low-temperature regime can be divided into two parts,  $\Theta_1 \geq \delta \gg \Theta$ , corresponding to the 3D case, and  $\Theta_1 \gg \Theta \gg \delta$ , corresponding to the quasi-2D case. When the interlayer coupling is not very weak, only the first part,  $\Theta_1 \geq \delta \gg \Theta$ , is retained.



**Figure 5.**  $\langle S^z \rangle_{A0}$ , the magnetizations of sublattice A, versus  $\delta$  with  $S_b = \frac{1}{2}$  at  $T = 0$  K for several values of  $S_a$ .



**Figure 6.**  $\langle S^z \rangle_{B0}$ , the minus magnetizations of sublattice B, versus  $\delta$ .

In the first low-temperature regime  $\Theta_1 \geq \delta \gg \Theta$ , the auxiliary functions  $n_A$  and  $n_B$  are calculated using the long-wavelength approximation in all directions of  $k$ . From (13) we have

$$n_A = n_{A0} + \frac{\zeta(\frac{3}{2})}{(1-\alpha)\sqrt{\delta}} \left( \frac{3(1-\alpha)\Theta}{8\pi \langle S^z \rangle_B} \right)^{3/2} \quad (16a)$$

$$n_B = n_{B0} + \frac{\alpha \zeta(\frac{3}{2})}{(1-\alpha)\sqrt{\delta}} \left( \frac{3(1-\alpha)\Theta}{8\pi \langle S^z \rangle_B} \right)^{3/2}. \quad (16b)$$

In the second low-temperature regime  $\Theta_1 \gg \Theta \gg \delta$ , using the long-wavelength approximation only in directions of  $k_x$  and  $k_y$  and integrating directly, we have

$$n_A = n_{A0} + \frac{3\Theta}{8\pi \langle S^z \rangle_B} \ln \left( \frac{3(1-\alpha)\Theta}{2 \langle S^z \rangle_B \delta} \right) \quad (17a)$$

$$n_B = n_{B0} + \frac{3\Theta}{8\pi \langle S^z \rangle_A} \ln \left( \frac{3(1-\alpha)\Theta}{2 \langle S^z \rangle_B \delta} \right), \quad (17b)$$

where  $n_{A0}$  and  $n_{B0}$  have been given by (14).

It is easy to see that the second term of the right-hand side in equations (16) and (17) is small. Expanding equation (12) at  $n_A = n_{A0}$  and  $n_B = n_{B0}$  and then using (16) and (17), we can obtain iterative solutions of the sublattice magnetizations in the corresponding low-temperature regime.

In the first low-temperature regime  $\Theta_1 \geq \delta \gg \Theta$ , we find that

$$\langle S^z \rangle_A = \langle S^z \rangle_{A0} - \frac{AS_a \zeta(\frac{3}{2})}{(S_a - S_b)\sqrt{\delta}} \left( \frac{3(S_a - S_b)\Theta}{8\pi S_a S_b} \right)^{3/2} \quad (18a)$$

$$\langle S^z \rangle_B = \langle S^z \rangle_{B0} - \frac{BS_b \zeta(\frac{3}{2})}{(S_a - S_b)\sqrt{\delta}} \left( \frac{3(S_a - S_b)\Theta}{8\pi S_a S_b} \right)^{3/2}. \quad (18b)$$

In the second low-temperature regime  $\Theta_1 \gg \Theta \gg \delta$ , we have

$$\langle S^z \rangle_A = \langle S^z \rangle_{A0} - \frac{3A\Theta}{8\pi S_b} \ln \left( \frac{3(S_a - S_b)\Theta}{2S_a S_b \delta} \right) \quad (19a)$$

$$\langle S^z \rangle_B = \langle S^z \rangle_{B0} - \frac{3B\Theta}{8\pi S_a} \ln \left( \frac{3(S_a - S_b)\Theta}{2S_a S_b \delta} \right) \quad (19b)$$

where  $\langle S^z \rangle_{A0}$  and  $\langle S^z \rangle_{B0}$  are the sublattice magnetizations at  $T = 0$  K, and

$$A = 1 - \frac{(2S_a + 1)^2 n_0^{2S_a}}{(1 + n_0)^{2S_a+1} - n_0^{2S_a+1}} \left( 1 - \frac{n_0[(1 + n_0)^{2S_a} - n_0^{2S_a}]}{(1 + n_0)^{2S_a+1} - n_0^{2S_a+1}} \right) \quad (20a)$$

$$B = 1 - \frac{(2S_b + 1)^2 n_0^{2S_b}}{(1 + n_0)^{2S_b+1} - n_0^{2S_b+1}} \left( 1 - \frac{n_0[(1 + n_0)^{2S_b} - n_0^{2S_b}]}{(1 + n_0)^{2S_b+1} - n_0^{2S_b+1}} \right) \quad (20b)$$

$$n_0 = -\frac{1}{2} + \frac{1}{2N} \sum_k \frac{S_a + S_b}{\sqrt{(S_a - S_b)^2 + 4S_a S_b(1 - \eta_k^2)}}. \quad (21)$$

From equations (18), we can see that, in the first low-temperature regime, the reduction in sublattice magnetizations follows the Bloch  $T^{3/2}$  law of 3D ferromagnets [15] while, in the second temperature regime, it follows the  $T \ln T$  law the same as in both quasi-2D ferromagnets and quasi-2D antiferromagnets [3, 10]. For weak interlayer coupling, with increase in temperature from zero, the temperature dependences of the sublattice magnetizations have a crossover from 3D to quasi-2D behaviour. At low temperatures, the reduction in magnetization due to the spin thermal excitation in sublattice B is smaller always than that of sublattice A. We can also see that the low-temperature behaviour of the sublattice magnetization obtained by the present method is very similar to that obtained by linear spin-wave theory [22], which indicates that the present method is suitable for use in dealing with layered ferrimagnets.

Finally, we discuss the behaviour of the sublattice magnetizations just below the Curie temperature  $\Theta_c$ . Because  $\langle S^z \rangle_A \rightarrow 0$  and  $\langle S^z \rangle_B \rightarrow 0$  as  $\Theta \rightarrow \Theta_c$ ; so we can expand the exponential of equation (13) in powers of  $E_A^\pm$  and  $E_B^\pm$ , giving

$$n_A = \frac{F_1 \Theta}{2\Theta_1 \langle S^z \rangle_B} - \frac{1}{2} + \frac{\Theta_1 \langle S^z \rangle_B}{6\Theta} \quad (22a)$$

$$n_B = \frac{F_1 \Theta}{2\Theta_1 \langle S^z \rangle_A} - \frac{1}{2} + \frac{\Theta_1 \langle S^z \rangle_A}{6\Theta} \quad (22b)$$

where

$$F_1 = \frac{1}{N} \sum_k \frac{1}{1 - \eta_k^2}. \quad (23)$$



Equation (22) shows that both  $n_A$  and  $n_B$  are large quantities. Expanding the right-hand side of equation (12) in powers of  $n_A - 1$  and  $n_B - 1$ , we have

$$(n_A + \frac{1}{2})\langle S^z \rangle_A = \frac{1}{3}S_a(S_a + 1) - \frac{3C_A\langle S^z \rangle_A^2}{2S_a(S_a + 1)} \quad (24a)$$

$$(n_B + \frac{1}{2})\langle S^z \rangle_B = \frac{1}{3}S_b(S_b + 1) - \frac{3C_B\langle S^z \rangle_B^2}{2S_b(S_b + 1)} \quad (24b)$$

where

$$C_A = (4S_a^2 + 4S_a - 3)/30 \quad (25a)$$

$$C_B = (4S_b^2 + 4S_b - 3)/30. \quad (25b)$$

Substituting (22) into (24), we obtain

$$\langle S^z \rangle_A^2 = \frac{\alpha S_a(S_a + 1)/3 - F_1\Theta/2\Theta_1}{3C_A/[2\alpha S_a(S_a + 1)] + \Theta_1/6\Theta} \quad (26a)$$

$$\langle S^z \rangle_B^2 = \frac{\alpha S_b(S_b + 1)/3 - F_1\Theta/2\Theta_1}{3\alpha C_B/[2S_b(S_b + 1)] + \Theta_1/6\Theta}. \quad (26b)$$

Considering  $\langle S^z \rangle_A = \langle S^z \rangle_B = 0$  at  $\Theta = \Theta_c$ , the numerator of equation (26) is equal to zero; we can find the limiting value of  $\alpha_c$  and the Curie temperature  $\Theta_c$  as follows:

$$\alpha_c = [S_a(S_a + 1)/S_b(S_b + 1)]^{1/2} \quad (27)$$

$$\Theta_c = \frac{2\Theta_1[S_a(S_a + 1)S_b(S_b + 1)]^{1/2}}{3F_1}. \quad (28)$$

Substituting (27) and (28) into (26), we obtain

$$\langle S^z \rangle_A = \frac{2S_a(S_a + 1)}{[3(F_1 + 6C_A)]^{1/2}} \left(1 - \frac{\Theta}{\Theta_c}\right)^{1/2} \quad (29a)$$

$$\langle S^z \rangle_B = \frac{2S_b(S_b + 1)}{[3(F_1 + 6C_B)]^{1/2}} \left(1 - \frac{\Theta}{\Theta_c}\right)^{1/2}. \quad (29b)$$

We see that, when the temperature approaches the Curie temperature, the asymptotic behaviour of the sublattice magnetization of the layered Heisenberg ferrimagnets is similar to that of Heisenberg ferromagnets and antiferromagnets; the critical exponent of sublattice magnetization is also  $\frac{1}{2}$ . We have calculated numerically the Curie temperature as a function of interlayer coupling strength using equation (28) and the results are plotted in figure 7; we see that the Curie temperature increases with increasing  $S_a$  for fixed  $\delta$  and also increases with increasing  $\delta$  for fixed  $S_a$ . As in [16], the weak-coupling limit, we can obtain

$$\Theta_c = \frac{4\pi[S_a(S_a + 1)S_b(S_b + 1)]^{1/2}}{\ln(32/\delta)}. \quad (30)$$

Equation (30) shows clearly that, when  $\delta = 0$ ,  $\Theta_c = 0$ , i.e. the pure 2D ferrimagnets do not have LRO at finite temperature as in pure 2D ferromagnets and antiferromagnets. The interlayer coupling also plays an important role in the stabilization of 3D LRO at a finite temperature for ferrimagnets.

#### 4. Summary

We have investigated a layered ferrimagnetic Heisenberg model by means of a double-time-temperature spin Green function. In the whole temperature regime the sublattice

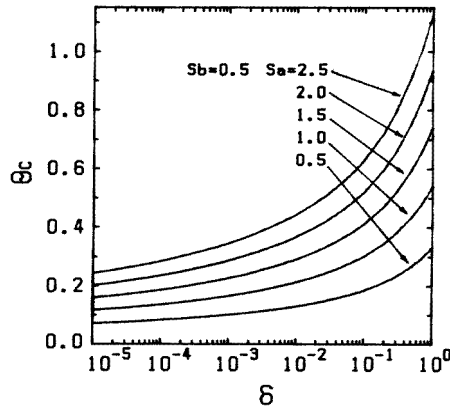


Figure 7. The reduced Curie temperature  $\Theta_c(k_B T_c/6J)$  versus  $\delta$ .

magnetizations are computed self-consistently for different spin quantum numbers  $S_a$  and interlayer coupling strengths  $\delta(= J_{\perp}/J)$ . The results show that the sublattice magnetizations decrease with increasing temperature  $T$ , with decreasing  $S_a$  and with decreasing  $\delta$ . At  $T = 0$  K, as in antiferromagnets, the zero-point quantum fluctuations of spin also exist in ferrimagnets. At low temperatures, for weak interlayer coupling, the asymptotic forms of sublattice magnetizations show that the temperature dependences of sublattice magnetizations undergo a crossover from 3D ( $T^{3/2}$ ) behaviour to quasi-2D ( $T \ln T$ ) behaviour as  $T$  increases from zero. When the temperature  $T$  approaches the Curie temperature, the asymptotic behaviours of sublattice magnetizations show that their critical exponents are all  $\frac{1}{2}$ . We finally give the calculation formula of the Curie temperature and its numerical results, as well as its asymptotic expression for weak interlayer coupling strength, which also show that the 2D ferrimagnets do not have LRO at non-zero temperature.

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